Roll No. | | | | | | | | | | | | | Total No. of Pages : 04

Total No. of Questions : 09

B.Tech. (Sem.–2nd) (2011 Batch)

ENGINEERING MATHEMATICS-II

Subject Code : BTAM-102

Paper ID : [A1111]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1.

(a) For what values of *a, b*, and *c* the matrix $\frac{1}{3}\begin{pmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{pmatrix}$ is an orthogonal matrix?

(b) Is the matrix $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ diagonalisable ? Give reasons.

(c) For what value of "a" the series $\sum_{n=1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$ converges.

(d) Separate $\tan^{-1}(x+iy)$ into imaginary parts.

(e) Find the modulus and argument of the complex number $(1 + i)^{1-i}$.

(f) Let $\sum_{n=1}^{\infty} a_n$ is convergent series of non-negative numbers. What can be said about the convergence of the series $\sum_{n=1}^{\infty} \frac{a_n}{n}$.

(g) Is the equation $(5x^3 + 12x^2 + 6y^2)dx + 6xy\ dy = 0$ exact or not. If not, find integrating factor which will make it exact.

(h) Find the general solution of the equation $\frac{dy}{dx} = \log\left(x\frac{dy}{dx} - y\right)$.

(i) Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$.

(j) Find the rank of the matrix $\begin{pmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{pmatrix}$.

SECTION B

2. (a) Find the complete solution of the differential $y'' + 2y' + y = x \cos x$.

(b) Use method of variation of parameters to find the general solution of the differential equation $y'' + 16y = 32 \sec 2x$. (4,4)

3. (a) Find the complete solution of the differential equation :

$$x^2 y'' + xy' + y = \log x.$$

by using operator method.

(b) Solve the following simultaneous differential equation

$$\frac{dx}{dt}+3y+4x=t,\ \frac{dy}{dt}+2x+5y=e^{t}$$ (3,5)

4. (a) Solve the differential equation $\frac{dy}{dx}=\cot y(1-x\cos y)$

(b) Find the solution of the equation $y=2px+\tan^{-1}xp^{2}$, where $p=\frac{dy}{dx}$.

(4,4)

5. In an L-C-R circuit, the charge q on a plate of a condenser is given by the differential equation

$$L\frac{d^{2}q}{dt^{2}}+R\frac{dq}{dt}+\frac{1}{C}q=E\sin pt$$

The circuit is tuned to resonance so that $p^2 = 1/LC$. If initially the current i and the charge q are zero, then show that for small values of R/L , the current i at any time t in the circuit is given by $(Et/2L)\sin pt$

(8)

SECTION–C

6. (a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$.

(b) Use the rank method to find the values of λ *and* μ. for which the system of equations

$x+y+z=6$; $x+2y+3z=10$; $x+2y+\lambda z=\mu$; has

(i) No solution

(ii) Unique solution

(iii) Infinitely many solutions .

(4,4)

7. (a) Test the convergence/divergence of the following series

(i) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$ (ii) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

(b) Test the convergence/divergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+\sqrt{n}} - \sqrt{n}\right)$$

(c) Discuss the convergence /divergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{2^{\left(n^2\right)}}$.

(4,2,2)

8. (a) For what values of ***X*** does the series

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\text{.........}$$

converges and diverges.

(b) Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$

(4,4)

9. (a) Use C + i S method to find the sum of the series :

$$\sin\alpha\cos\alpha+\sin^2\alpha\cos 2\alpha+\sin^3\alpha\cos 3\alpha+\text{.......}\infty.$$

(b) If $\sin^{-1}(u+iv)=\alpha+i\beta$, then prove that $\sin^2\alpha$ *and* $\cosh^2\beta$ are the roots of the equation $x^2 - x(1+u^2+v^2)+u^2=0.$ (4,4)